

Constraining the Lattice Fluid Dark Energy from SNe Ia, BAO and OHD

Xiaoxian Duan,* Yichao Li,[†] and Changjun Gao[‡]

The National Astronomical Observatories, Chinese Academy of Sciences, China

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Sanchez and Lacombe have ever developed a lattice fluid theory based on a well-defined statistical mechanical model. Taking the lattice fluid as a candidate of dark energy, we investigate the cosmic evolution of this fluid. Using the combined observational data of Type Ia Supernova (SNe Ia), Baryon Acoustic Oscillations (BAO) and Observational Hubble Data (OHD), we find the best fit value of the parameter in the model, $A = -0.3^{+0.1}_{-0.1}$. Then the cosmological implications of the model are presented.

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I. INTRODUCTION

Increasing amount of cosmological observations have provided convincing evidences that our Universe is undergoing a late time acceleration. The observations of Type Ia Supernova (SNe Ia) [1, 2], Cosmic Microwave Background (CMB) [3–5], the Baryon Acoustic Oscillations (BAO) [6–8] and the Hubble data based on differential ages of the galaxies (OHD) [9] definitely reveal the accelerating expansion of the Universe. Then how to explain the acceleration of the Universe? The simplest explanation is the cosmological constant (CC) which is raised by Einstein in General Relativity. The cosmological constant could fit the observations very well. However, it suffers from two fundamental problems: the fine tuning problem and the coincidence problem [10]. The fine tuning problem is the following. The cosmological constant is also the vacuum energy in quantum field theory. Then the estimated value of the vacuum energy $\rho \simeq \rho_p$ (where ρ_p is the Planck density) is greater than the observed value $\rho \simeq 10^{-123} \rho_p$ by 123 orders of magnitude. The coincidence problem is the following. The densities of the dark energy and matter evolve differently as the Universe expands. But they are comparable in the present-day Universe. It is a great coincidence if there are no any internal connections between them.

Therefore, a lot of dynamical dark energy models have been explored. They are mainly classified into several types: scalar field theories, modified gravity theories, phenomenological models, and fluid dark energy models. The first type includes quintessence [11–14], phantom [15], K-essence [16–18], tachyon [19, 20], quintom [21]. The second type includes $f(R)$ theory [22–26], DGP theory [27] and so on. The third type includes holographic dark energy [28], the agegraphic dark energy [29] and so on. Last but not the least, there are fluid dark energy

models mainly cover the chaplygin gas [30–38] and the Van der Waals gas [39–41].

Kamenshchik *et al.* studied the standard Chaplygin gas [30] with the equation of state $p = -\frac{A}{\rho}$ (A is a positive constant). The standard Chaplygin gas could behave as a pressureless dust at early times and as a cosmological constant at late times in the evolution of the Universe. But it has difficulty in explaining the CMB anisotropic observations. The situation could be alleviated in the generalized Chaplygin gas [31] which has the equation of state $p = -\frac{A}{\rho^\alpha}$ with α severely constrained, $0 \leq \alpha < 0.2$. In general, Benaoum studied a new equation of state $p = A\rho - \frac{B}{\rho^n}$ ($n \geq 1$, A and B are positive constants). It could interpolate between standard fluids at high energy densities and Chaplygin gas fluids at low energy densities [32]. On the other hand, Ref. [40, 41] investigated the possibility of Van der Waals fluid as both the dark matter and dark energy, and Ref. [39] studied the evolution of the Universe filled with a mixture of van der Waals fluid and dark energy. The lattice fluid dark energy (LFDE) investigated in this paper falls into this last type.

The equation of state for lattice fluid is derived by Sanchez and Lacombe [42]. One find that the lattice fluid model could perfectly describe the thermodynamic properties of a wide variety of fluids. On the other hand, the four laws of black hole mechanics, which are analogous to those of thermodynamics, were originally derived from the classical Einstein equation [43]. Until the discovery of the quantum Hawking radiation [44], one recognize that the analogy is in fact an identity. So the nature of space, time and matter is closely related to the thermodynamics. By turning the logic around, we started from the thermoplastics in order to seek for the candidate of dark energy. We find that the lattice fluid could behave as a fluid with negative pressure which is exactly the property of dark energy. Therefore we constrain the LFDE from the observations of SNe Ia, BAO and OHD.

The paper is organized into four sections. In section II, we derive the equation for the cosmic evolution of LFDE in the background of Friedmann-Robertson-Walker (FRW) Universe. In section III, using the observational data of SNe Ia, BAO and OHD, we seek for

*Electronic address: duan@bao.ac.cn

[†]Electronic address: ycli@bao.ac.cn

[‡]Electronic address: gaocj@bao.ac.cn

the best fit value of the parameter in the model. In section IV, we present the main results: the best fit value of the dimensionless parameter A , the evolution of deceleration parameter q , the statefinder parameters r , s with the redshift z . Section V gives the conclusion and discussion. We shall use the units in which the speed of light c is set to unity.

II. LATTICE FLUID DARK ENERGY

The equation of state for LFDE can be written as [42]

$$p_X = -\frac{\rho_X^2}{\rho_0} - A\rho_0 \left[\ln(1 - \frac{\rho_X}{\rho_0}) + \frac{\rho_X}{\rho_0} \right], \quad (1)$$

where ρ_X and p_X are the energy density and pressure of the fluid, respectively. A is a dimensionless constant and ρ_0 is the present-day cosmic energy density defined by $\rho_0 = \frac{3H_0^2}{8\pi}$ with H_0 the present-day Hubble parameter.

Now we study the evolution of the Universe filled with matter (includes baryon matter and dark matter) and LFDE. The metric of FRW Universe is given by:

$$ds^2 = -dt^2 + a^2(t) \left[\frac{dr^2}{1 - Kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right], \quad (2)$$

where $a(t)$ is the scale factor and $K = +1, 0, -1$ describe the topology of the Universe which correspond to closed, flat and open Universe, respectively.

The scale factor evolves according to the Friedmann equation:

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_{total} - \frac{K}{a^2}, \quad (3)$$

where ρ_{total} is the total energy density of the Universe. Observations reveal that the Universe is highly flat in space. So we put $K = 0$ in the following. For the matter-plus-dark energy dominated Universe, we can safely neglect the contribution of relativistic matter. Then the Friedmann equation can be written as

$$3H^2 = 8\pi G \left(\frac{\rho_m}{a^3} + \rho_X \right), \quad (4)$$

where $\frac{\rho_m}{a^3}$ is the energy density of matter and ρ_m is the energy density of matter in the present-day Universe.

In order to obtain ρ_X , we substitute the equation of state for LFDE Eq. (1) into the energy conservation equation:

$$\frac{d\rho_X}{da} + \frac{3}{a}(\rho_X + p_X) = 0. \quad (5)$$

Keeping in mind the relation $\frac{1}{a} = 1 + z$ and defining $\Omega_X \equiv \frac{\rho_X}{\rho_0}$, we obtain the energy conservation equation as follows

$$\frac{d\Omega_X}{dz} - \frac{3}{1+z} \{ \Omega_X - \Omega_X^2 - A [\ln(1 - \Omega_X) + \Omega_X] \} = 0. \quad (6)$$

This ordinary differential equation could not be solved analytically. So we are going to study the numerical solution by using the observational data from Supernovae Ia, Baryonic Acoustic Oscillations and the Observational Hubble Data.

III. CONSTRAINTS FROM SNE IA, BAO AND OHD

A. Supernovae Ia

Supernovae Ia are generally believed to have homogeneous intrinsic luminosity of peak magnitude. So Supernovae Ia are usually known as standard candles which could be used to measure the expansion history of the Universe. The analysis of their distance modulus versus redshift could provide direct evidence for the acceleration of the Universe and the analysis also put a constraint on dark energy models.

Therefore, with the observation of SNe Ia data, we could obtain the best fit value of the parameter in our model. In order to constrain the equation of state for the LFDE, we take the Constitution set with 397 SNe Ia in terms of the distance modulus $\mu_{obs}(z)$ compiled in Table 1 of Ref. [45].

For the spatially flat Universe, the Friedmann equation Eq. (3) could be written as

$$H(z) = H_0 \sqrt{\Omega_m (1+z)^3 + \Omega_X}, \quad (7)$$

where $\Omega_m = \rho_m/\rho_0$. The luminosity distance d_L of SNe Ia is defined by

$$d_L(z) = \frac{c(1+z)}{H_0} F(z). \quad (8)$$

The function $F(z)$ is defined by

$$F(z) = \int_0^z \frac{1}{E(z')} dz', \quad (9)$$

with

$$E(z) = \frac{H(z)}{H_0} = \sqrt{\Omega_m (1+z)^3 + \Omega_X}. \quad (10)$$

The theoretical distance modulus is defined by

$$\mu_{th}(z) = m - M = 5 \log \frac{d_L}{10\text{pc}} = 42.39 + 5 \log \frac{1+z}{h} F(z), \quad (11)$$

where m and M are the apparent and absolute magnitudes, respectively. On the other hand, the observational distance modulus is give by

$$\mu_{obs} = m - M + \alpha(s-1) - \beta c, \quad (12)$$

where α and β are dimensionless parameters, s and c are shape parameter and color parameter derived from the fitting to light curves, respectively.

Then we could calculate χ^2

$$\chi_{SNIa}^2 = \sum_i^{397} \frac{(\mu_{th_i} - \mu_{obs_i})^2}{\sigma_i^2(\mu_B) + \sigma_{int_i}^2}, \quad (13)$$

where $\sigma(\mu_B)$ is the observational variable, which depends on α and β . $\sigma(\mu_B)$ also includes the contribution of peculiar velocity, 400 km/s . σ_{int} is the intrinsic dispersion of SNe absolute magnitudes.

B. Baryonic Acoustic Oscillations (BAO)

The Baryonic Acoustic Oscillations signatures in the large-scale clustering of galaxies could act as additional tests for cosmology, because the acoustic oscillations in the relativistic plasma of the early Universe could be imprinted onto the late-time power spectrum of the non-relativistic matter [46]. Therefore it could be used to put an additional constraint on dark energy models.

The BAO relevant distance measure is modelled by volume distance, which is defined as

$$D_V(z) = [D_M^2(z)z/H(z)]^{1/3}, \quad (14)$$

where $H(z)$ is the Hubble parameter and $D_M(z) = \int_0^z \frac{1}{H(z')} dz'$ is the comoving angular diameter distance.

Eisenstein *et al.* [6] studied the spectroscopic sample of 46748 luminous red galaxies from the Sloan Digital Sky Survey. They found that the combination $D_V(0.35)\sqrt{\Omega_m h^2}$ has no dependence on the Hubble constant H_0 because $D_V(0.35)$ is proportional to H_0^{-1} . And the combination is well constrained by their data.

They measured

$$A_{obs} \equiv D_V(0.35) \frac{\sqrt{\Omega_m H_0^2}}{0.35c} = 0.469 \pm 0.017 (3.6\%). \quad (15)$$

For a flat Universe, we have

$$A_{th} = \sqrt{\Omega_m} E(z_1)^{-1/3} \left[\frac{1}{z_1} \int_0^z \frac{1}{E(z')} dz' \right], \quad (16)$$

where $z_1 = 0.35$. Therefore χ_{BAO}^2 is

$$\chi_{BAO}^2 = \frac{(A_{th} - A_{obs})^2}{\sigma^2}. \quad (17)$$

C. The Observational Hubble Data (OHD)

Relative Galaxy Ages can also be used to constrain cosmological parameters [9]. Given the measurement of the age difference of two passively-evolving galaxies formed nearly at the same time, δt , and the small redshift interval δz by which they are separated, the ratio $\frac{\delta z}{\delta t}$ could be calculated. Then we can infer the derivative: $\frac{dz}{dt}$.

The quantity measured in the method above is directly related to the Hubble parameter:

$$H(z)_{obs} = -\frac{1}{1+z} \frac{dz}{dt}. \quad (18)$$

In this paper, we take the observational data in Table V in Ref. [47] from Gemini Deep Survey (GDDS), SPICES and VVDS, and Keck Observations. We can obtain the theoretical value of $H(z)$ from the Friedmann equation Eq. (7).

Therefore χ^2 for Hubble data is

$$\chi_{OHD}^2 = \sum_i^{12} \frac{(H_{th_i} - H_{obs_i})^2}{\sigma_i^2}. \quad (19)$$

Now we have obtained the χ^2 of SNIa, BAO and H_0 . Then the total χ^2 is given by:

$$\chi_{tot}^2 = \chi_{SNIa}^2 + \chi_{BAO}^2 + \chi_{OHD}^2. \quad (20)$$

IV. RESULTS

A. Best Fit Value of A

Using the joint constraints of SNe Ia, BAO, and OHD, we get the best fit value of A : $-0.3^{+0.1}_{-0.1}$. In Fig. 1, we plot the distance modulus for the model and that for the observational data. In Fig. 2, we plot the evolution

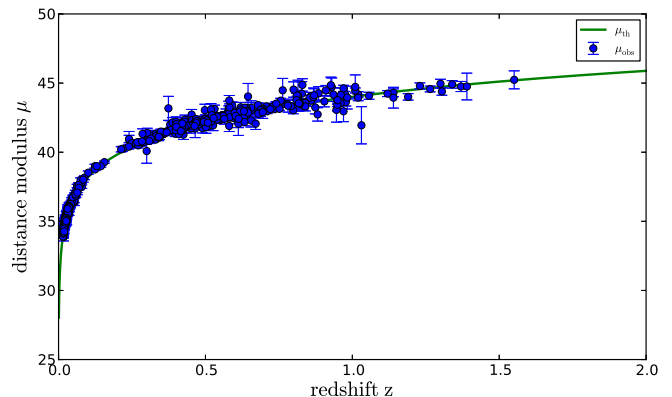


FIG. 1: The observational distance modulus and the theoretical distance modulus when $A = -0.3$. The dots with error bars are for μ_{obs} of the 397 observed SNe Ia data. The solid line is for the theoretical distance modulus that calculated in a Universe filled with matter and LFDE.

of the Hubble parameter with redshifts. It is consistent with the observational data very well.

In Fig. 3, we plot the equation of state, $w = p_X/\rho_X$ for the lattice fluid. It is obvious that the equation of state of LFDE is approaching to -1 at the redshifts greater than

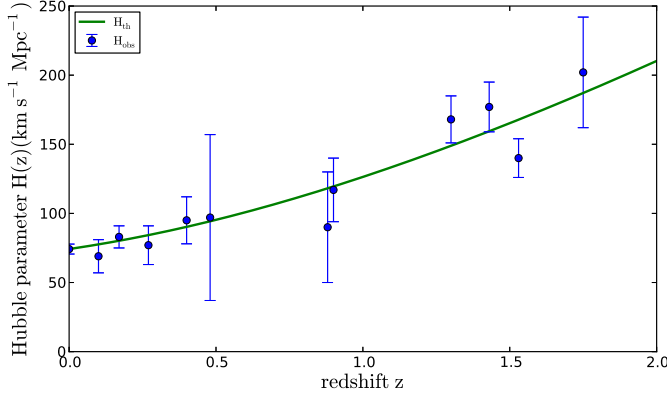


FIG. 2: The Observational Hubble Data and the theoretical Hubble parameter when $A = -0.3$. The dots with errorbars are the 12 observational Hubble data. The solid line is for the theoretical Hubble parameter that calculated in a Universe filled with dark matter and LFDE.

2. In other words, the lattice fluid behaves as a cosmological constant at the redshifts greater than 2. Therefore, we conclude that the structure formation history would not be modified by this lattice fluid.

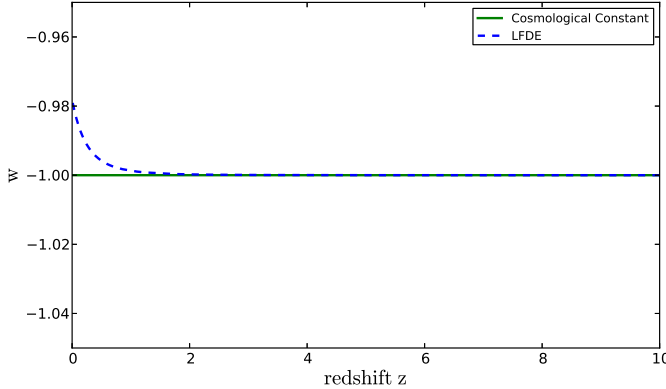


FIG. 3: The evolution of the equation of state $w = \frac{p_X}{\rho_X}$. The blue dashed line is the equation of state for the cosmological constant. The green solid line is the equation of state for LFDE. When $z > 2.0$, w is approaching -1 .

B. Age of the Universe

The age of the Universe could be calculated from the Friedmann equation, Eq. (3). The duration of radiation dominated epoch is very short compared to the total history of the Universe. So in order to calculate the age of the Universe, we could simply consider the two components of the cosmic matter sources: matter (including

dark matter and baryon matter) and dark energy. Using Eq. (7) and taking account of the relation $1 + z = \frac{1}{a}$, one find the age of the Universe

$$\begin{aligned} t_0 &= \int_0^{t_0} dt = \int_0^\infty \frac{dz}{H(1+z)} \\ &= \int_0^\infty \frac{dz}{H_0(1+z)\sqrt{\Omega_m(1+z)^3 + \Omega_X}}. \end{aligned} \quad (21)$$

Then the age of the Universe is found to be 13.5 Gyr.

C. The Deceleration Parameter and the Statefinder

The deceleration parameter q is defined by

$$q \equiv -\frac{\ddot{a}}{aH^2}. \quad (22)$$

By using of the Friedmann equation, Eq. (3), and the energy conservation equation

$$\dot{\rho}_{total} = -3H(\rho_{total} + p_{total}), \quad (23)$$

we can rewrite the deceleration parameter as follows [48]

$$q = \frac{1}{2} + \frac{3}{2} \frac{p_{total}}{\rho_{total}}, \quad (24)$$

where p_{total} and ρ_{total} are the total pressure and total energy density of the Universe, respectively. Since the total energy density and pressure are contributed by dust component and LFDE component, Eq. (24) is actually

$$q = \frac{1}{2} + \frac{3}{2} \frac{p_d + p_X}{\rho_d + \rho_X} = \frac{1}{2} + \frac{3}{2} \frac{p_X}{\rho_d + \rho_X}. \quad (25)$$

Keeping in mind the equation of state of LFDE, $w = p_X/\rho_X$, and $\rho_d = \rho_m(1+z)^3$, we get

$$q = \frac{1}{2} + \frac{3}{2} \frac{w}{1 + \frac{\Omega_m}{\Omega_X}(1+z)^3}. \quad (26)$$

We have plotted the evolution of deceleration parameter q with redshifts in Fig. 4. We find it has nearly the same behavior as the Λ CDM model.

The statefinder is introduced by Sahni et al [50]. The definitions are as follows:

$$r \equiv \frac{\ddot{a}}{aH^3}, \quad s \equiv \frac{r-1}{3(q-\frac{1}{2})}. \quad (27)$$

Using the Friedmann equation, Eq. (3), and the energy conservation equation, Eq. (23), it is easy to find

$$\begin{aligned} r &= 1 - \frac{3\dot{p}}{2\rho\sqrt{\rho}} \sqrt{\frac{3}{8\pi G}}, \\ s &= -\frac{\dot{p}}{3\rho\sqrt{\rho}} \sqrt{\frac{3}{8\pi G}}, \end{aligned} \quad (28)$$

where ρ and p stands for the total energy density and total pressure, respectively. For a two-component matter sources, Eqs. (28) take the form

$$\begin{aligned} r &= 1 + \frac{9}{2(\rho_1 + \rho_2)} \left[\frac{\partial p_1}{\partial \rho_1} (\rho_1 + p_1) + \frac{\partial p_2}{\partial \rho_2} (\rho_2 + p_2) \right], \\ s &= \frac{1}{p_1 + p_2} \left[\frac{\partial p_1}{\partial \rho_1} (\rho_1 + p_1) + \frac{\partial p_2}{\partial \rho_2} (\rho_2 + p_2) \right]. \end{aligned} \quad (29)$$

For our case, we have $p_1 = p_d = 0$. Then Eqs. (29) become

$$\begin{aligned} r &= 1 + \frac{9}{2} \frac{\rho_X + p_X}{\rho_X + \rho_d} \frac{\partial p_X}{\partial \rho_X}, \\ s &= \frac{\rho_X + p_X}{p_X} \frac{\partial p_X}{\partial \rho_X}. \end{aligned} \quad (30)$$

Finally, from the equation of state for LFDE, Eq. (1), we have

$$\frac{\partial p_X}{\partial \rho_X} = -2\Omega_X - A \left(-\frac{1}{1 - \Omega_X} + 1 \right). \quad (31)$$

The evolution of the statefinder r , s are shown in Fig. 5 and Fig. 6. They show that when the redshifts are greater than 2.0, they have almost the same r and s as the cosmological constant. On the other hand, with the decreasing of redshifts, r and s evolve to the present values: $r = 0.762$, $s = 0.075$.

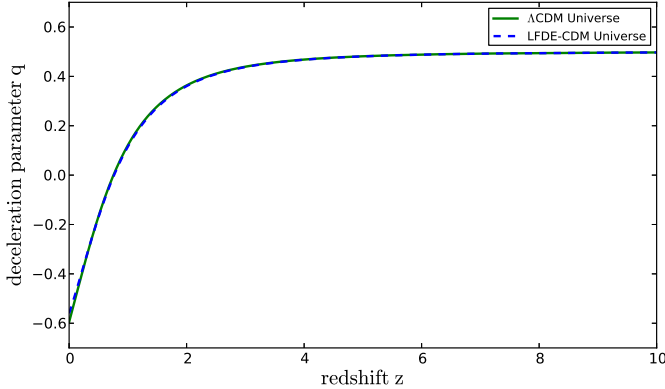


FIG. 4: The evolution of the deceleration parameter q with the redshifts. The green solid line is the deceleration parameter of Λ CDM Universe. The blue dashed line is for the LFDE model.

V. CONCLUSIONS AND DISCUSSIONS

The remarkable discovery of the quantum Hawking radiation [44] reveals that the nature of space, time and matter is closely related to the statistic thermodynamics.

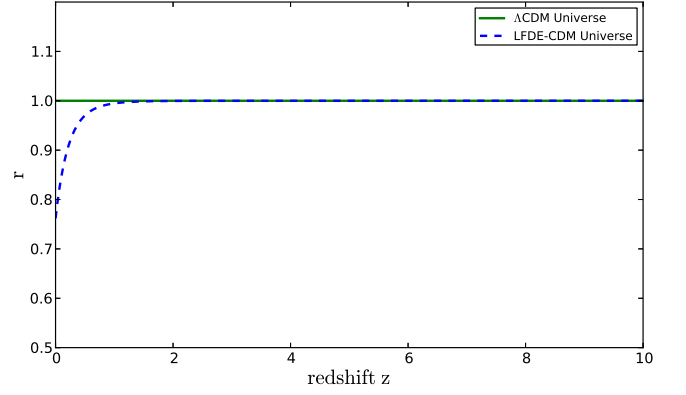


FIG. 5: The evolution of r with the redshifts. The green solid line is for the Λ CDM model. The blue dashed line is for the LFDE model.

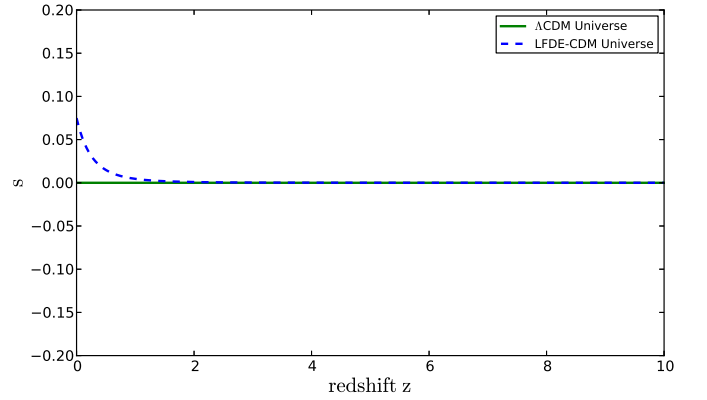


FIG. 6: The evolution of s with the redshifts. The green solid line is for the Λ CDM model. The blue dashed line is for the LFDE model.

By turning the logic around, we started from the result of static thermoplastics in order to seek for the candidate of dark energy. Sanchez and Lacombe have ever developed a lattice fluid theory based on a well-defined statistical mechanical model [42]. One find that the lattice fluid model is so interesting that it could perfectly describe the thermodynamic properties of a wide variety of fluids. Motivated by this point, we explore the possibility of the lattice fluid as the candidate of dark energy.

We constrain the model with current cosmological observational data, including observational data of SNe Ia from the Constitution set, BAO from SDSS, and Observational Hubble Data from GDDS, SPICES, VVDS and Keck observations. We find the best fit value of parameter A : $A = -0.3^{+0.1}_{-0.1}$. Taking the best value of A , we investigate the comic implications of the model. We find the equation of state and the statefinder of the lattice fluid are almost the same as the Λ CDM model at the

redshifts greater than 2. So the structure formation history would not be modified by this fluid. For the present-day Universe, we have the equation of state, $w = -0.968$ which is consistent with many other astronomical observations.

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